Exercise II

- 1. Sketch the graph of a non-constant function f(x) with domain [1,5] and with f(1) = f(5).
- 2. Sketch the graph of a non-constant function f(x) with domain [1,5], with f(1) = f(5) and f'(x) = 0 for two values of $x \in (1, 5)$.
- 3. Sketch the graph of a function f(x) with domain [1,5], with f(1) = f(5) and such that $f'(x) \neq 0$ for any $x \in (1,5)$.
- 4. Determine whether each of the following sets is bounded above or not, bounded below or not , bounded or not.If the set is bounded above determine the lub. If the set is bounded below determine the glb. Determine whether each set has a maximum and/or a minimum element or not.

(i)
$$[-10, 10]$$
, (ii) $(-\infty, 0]$, (iii) $(10, \infty)$, (iv) \mathbb{Q} , (v) $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$,
(vi) $\left\{\frac{1}{x} : x \in \mathbb{R}, x > 0\right\}$, (vii) $\{x \in Q : x^2 \le 2\}$

5. Sketch the graph of each of the following functions. State whether each function is bounded above or not, bounded below or not, bounded or not.

Determine whether each function has a maximum and/or a minimum value or not.

Determine the domain and range of each function.

(i)
$$f(x) = x^{2}$$

(ii) $f(x) = x^{3}$
(iii) $f(x) = x^{2}, x \in [-2, 2]$
(iv) $f(x) = x^{2}, x \in (-2, 2)$
(v) $f(x) = x^{3}, x \in (-2, 2]$
(vi) $f(x) = x^{3}, x \in (-2, 2)$
(vii) $f(x) = \begin{cases} x^{2}, x \in (-2, 2) \\ 3, x = -2 \\ 3, x = 2 \end{cases}$
(viii) $f(x) = \begin{cases} \frac{1}{x}, x \in (0, 1] \\ 2, x = 0 \end{cases}$
6. (i) Sketch the graph of $\{\frac{n}{n+1}\}$.
Find all values of $n \in \mathbb{N}$ such that
(ii) Find all $n \in \mathbb{N}$ such that $\left|\frac{n}{n+1} - \frac{n}{n+1}\right|$

Find all values of
$$n \in \mathbb{N}$$
 such that $\left|\frac{n}{n+1} - 1\right| < 0.01$.
(ii) Find all $n \in \mathbb{N}$ such that $\left|\frac{n}{n+1} - 1\right| < 0.001$.
(iii) Let $\epsilon > 0$. Find all $n \in \mathbb{N}$ such that $\left|\frac{n}{n+1} - 1\right| < \epsilon$.

(iv) Sketch the graph of
$$\left\{\frac{(-1)^n}{n}\right\}$$
.
Find all $n \in \mathbb{N}$ such that $\left|\frac{(-1)^n}{n}\right| < 0.0001$.