## Exercise II

1. Sketch the graph of a non-constant function $f(x)$ with domain $[1,5]$ and with $f(1)=f(5)$.
2. Sketch the graph of a non-constant function $f(x)$ with domain [1,5], with $f(1)=f(5)$ and $f^{\prime}(x)=0$ for two values of $x \in(1,5)$.
3. Sketch the graph of a function $f(x)$ with domain $[1,5]$, with $f(1)=f(5)$ and such that $f^{\prime}(x) \neq 0$ for any $x \in(1,5)$.
4. Determine whether each of the following sets is bounded above or not, bounded below or not, bounded or not.
If the set is bounded above determine the lub. If the set is bounded below determine the glb. Determine whether each set has a maximum and/or a minimum element or not.
(i) $[-10,10]$,
(ii) $(-\infty, 0]$,
(iii) $(10, \infty)$,
(iv) $\mathbb{Q}$
(v) $\left\{\frac{1}{n}: n \in \mathbb{N}\right\}$,
(vi) $\left\{\frac{1}{x}: x \in \mathbb{R}, x>0\right\}, \quad$ (vii) $\left\{x \in Q: x^{2} \leq 2\right\}$
5. Sketch the graph of each of the following functions.

State whether each function is bounded above or not, bounded below or not, bounded or not.
Determine whether each function has a maximum and/or a minimum value or not.
Determine the domain and range of each function.
(i) $f(x)=x^{2}$
(ii) $f(x)=x^{3}$
(iii) $f(x)=x^{2}, \quad x \in[-2,2]$
(iv) $f(x)=x^{2}, \quad x \in(-2,2)$
(v) $f(x)=x^{3}, \quad x \in(-2,2]$
(vi) $f(x)=x^{3}, \quad x \in(-2,2)$
(vii) $f(x)= \begin{cases}x^{2}, & x \in(-2,2) \\ 3, & x=-2 \\ 3, & x=2\end{cases}$
(viii) $f(x)= \begin{cases}\frac{1}{x}, & x \in(0,1] \\ 2, & x=0\end{cases}$
6. (i) Sketch the graph of $\left\{\frac{n}{n+1}\right\}$.

Find all values of $n \in \mathbb{N}$ such that $\left|\frac{n}{n+1}-1\right|<0.01$.
(ii) Find all $n \in \mathbb{N}$ such that $\left|\frac{n}{n+1}-1\right|<0.001$.
(iii) Let $\epsilon>0$. Find all $n \in \mathbb{N}$ such that $\left|\frac{n}{n+1}-1\right|<\epsilon$.
(iv) Sketch the graph of $\left\{\frac{(-1)^{n}}{n}\right\}$.

Find all $n \in \mathbb{N}$ such that $\left|\frac{(-1)^{n}}{n}\right|<0.0001$.

